INTERPOLATION FORMULAS FOR MAXWELL VISCOSITY OF CERTAIN METALS AS A FUNCTION OF SHEAR-STRAIN INTENSITY AND TEMPERATURE

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UDC 539.3+536.2

The purpose of this study is the construction of interpolation formulas for the dependence of Maxwell viscosity, a quantity which is the reciprocal of shear-strain relaxation time τ, on shear-strain intensity and temperature for several metals: iron, aluminum, copper, and lead. This function was interpolated in various temperature and deformation velocity $\dot{\varepsilon}$ ranges in accordance with available experimental data for iron ($0 \leq \dot{\varepsilon} \leq 10^7$ sec^{-1}, $200° \leq T \leq 1500°$); aluminum ($0 \leq \dot{\varepsilon} \leq 10^7$ sec^{-1}, $300° \leq T \leq 900°$); copper ($0 \leq \dot{\varepsilon} \leq 10^5$ sec^{-1}, $300° \leq T \leq 1300°$); lead ($0 \leq \dot{\varepsilon} \leq 10^6$ sec^{-1}, $90° \leq T \leq 400°$); temperatures in °K.

A system of differential equations for the nonlinear theory of elasticity in isotropic media was formulated in [1]. Such a medium is characterized by an equation of state $E(I_1, I_2, I_3, S)$ for the dependence of the internal energy per unit mass on the invariants I_1, I_2, I_3 of the deformation tensor, the entropy per unit mass S, and the "maxwell viscosity" χ.

To complete the system of differential equations [1] it is necessary to know the internal energy and stress relaxation time τ as functions of the parameters of the medium. Interpolation formulas for the internal energy E (equation of state) were presented in [2]. The equations developed in this present study together with the results of [2] complete the system of equations of [1].

The literature has examined the dependence of the dynamic creep limit σ_g and the yield strength σ_B on deformation rate $\dot{\varepsilon}$ (see section 3). Since the dynamic stresses σ_g and σ_B are of the same order [3] and define the intensity of shear strain in the medium, while $\dot{\varepsilon}$ defines the rate of plastic deformation occurring in the medium, we will assume that the characteristic time of plastic relaxation processes will be determined by $\tau = \dot{\varepsilon}^{-1}$, $\dot{\varepsilon} = \dot{\varepsilon}(\sigma, T)$, where σ is the shear strain intensity.

1. We present below formulas for the quantity $\chi = \tau^{-1}$ in sec^{-1}, the Maxwell viscosity

$$\chi = \chi_0 \left(\frac{\sigma}{\rho_0 c_0^2} q\right)^{n(T)} \exp\left(-\frac{U(\sigma, T)}{RT/\mu}\right), \quad \chi_0 = 1$$
$$n(T) = \left[n_0\left(\frac{T}{\theta_0} - n_1\right)^3 + n_2\right]^{-1}, \quad U(\sigma, T) = c_0^2 [n(T) F(T) \pm \Phi(\sigma)] \tag{1.1}$$
$$F(T) = (F_0 - F_1 T/\theta_0)\, T/\theta_0, \quad \Phi(\sigma) = \Phi_0 [\varphi(\sigma) - \sqrt{\varphi^2(\sigma) + \Phi_1}]$$
$$\varphi(\sigma) = \varphi_0 \ln [(\sigma / \rho_0 c_0^2) q] + \varphi_2, \quad \sigma = \{^1/_2 [\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]\}^{1/2}$$

Here T is temperature; μ, molecular weight; θ_0, Debye temperature; ρ_0 and c_0, density and velocity of longitudinal waves under normal conditions; σ, shear strain intensity, where σ_1, σ_2, σ_3 are the main stresses; $R = 8.31 \cdot 10^7$ erg/deg·mole, the universal gas constant; the values of the dimensional quantities ρ_0, c_0, θ_0, and μ are presented in Table 1, and the interpolation constants q, n_0, n_1, n_2, F_0, F_1, Φ_0, Φ_1, φ_0, φ_1 are tabulated in Table 2.

The formula for U contains a minus sign in the case of lead, a plus sign for the remaining metals.

Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 4, pp. 114-118, July-August, 1974. Original article submitted November 21, 1973.

TABLE 1

	g/cm³	kg/sec	θ_0, °K	g/mole
Fe	7.84	5.694	420	55.85
Al	2.785	6.125	390	26.98
Cu	8.90	4.651	315	63.54
Pb	11.34	2.151	88	207.21

TABLE 2

	Fe	Al	Cu	Pb
q	$2.6 \cdot 10^4$	$1.06 \cdot 10^4$	$1.97 \cdot 10^4$	$0.535 \cdot 10^4$
n_0	0.0434	0.0462	0.0202	0.00804
n_1	1.545	2.57	0.955	0
n_2	0.03	0.01	0.035	0.01
F_0	$7.12 \cdot 10^{-3}$	$1.18 \cdot 10^{-2}$	$7.15 \cdot 10^{-3}$	$1.6 \cdot 10^{-3}$
F_1	$1.89 \cdot 10^{-3}$	$4.77 \cdot 10^{-3}$	$0.99 \cdot 10^{-3}$	$0.294 \cdot 10^{-3}$
Φ_0	$1.37 \cdot 10^{-3}$	$3.19 \cdot 10^{-3}$	0	$2.6 \cdot 10^{-3}$
Φ_1	14.15	53.1	0	10.15
φ_0	7.85	21.25	0	14.9
φ_1	—32.5	—59.7	0	—91.0

TABLE 3

	kg/mm²	sec^{-1}	m_0	m_1
Fe	112	$2.515 \cdot 10^7$	$7.75 \cdot 10^{-2}$	1.023
Al	13	$3.98 \cdot 10^6$	$12.6 \cdot 10^{-2}$	0.919
Cu	26	$1.0 \cdot 10^5$	$7.4 \cdot 10^{-2}$	1.121
Pb	—	$3.165 \cdot 10^5$	$3.96 \cdot 10^{-2}$	1.023

The form of the interpolation formulas was chosen commencing from the general form of the functions $\dot{\varepsilon} = \dot{\varepsilon}(\sigma, T)$ presented in [4, 5]

$$\dot{\varepsilon} = \dot{\varepsilon}_0 (\sigma / \sigma_0)^m \exp(-U\mu / RT) \qquad (1.2)$$

where $\dot{\varepsilon}_0$ and σ_0 are certain interpolation coefficients.

2. In [6] experimental data were presented for iron, aluminum, and copper in a study of the function $\sigma_B = \sigma_B(\dot{\varepsilon}, T)$ in the ranges $1100° K \leq T \leq 1500° K$, $0 \leq \dot{\varepsilon} \leq 10^3 sec^{-1}$, $500° K \leq T \leq 900° K$, $0 \leq \dot{\varepsilon} \leq 10^3 sec^{-1}$, $700° K \leq T \leq 1300° K$, $0 \leq \dot{\varepsilon} \leq 10^3 sec^{-1}$, respectively.

The experimental results were interpolated with the formula

$$\sigma_B = \sigma_0 (\dot{\varepsilon} / \dot{\varepsilon}_0)^{\beta(T)}, \quad \beta(T) = m_0 (T / \theta_0 - m_1) \qquad (2.1)$$

The values of the constants $\dot{\varepsilon}_0$, σ_0, m_0, m_1 are presented in Table 3. The basic material studied in [6] was lead for the hardness of which H_k an interpolation formula analogous to Eq. (2.1) was constructed. The temperature and deformation rate in the experiments of [6] was

$$300° K \leqslant T \leqslant 500° K, \quad 0 \leqslant \dot{\varepsilon} \leqslant 10^3 \ sec^{-1}$$

It was established from the coincidence of the functions for hardness H_k and strength σ_B that in the given deformation rate range σ_B and H_k are proportional. We note that Eq. (2.1) is valid for $T > \theta_0$. Since for Fe, Al, and Cu $\theta_0 > 300°$, supplementary information on the temperature function $\sigma_g = \sigma_g(\dot{\varepsilon}, T)$ for $T < \theta_0$, is necessary. For conversion of the H_k value to σ_B for lead it is necessary to know the values of the proportionality coefficient. These data were presented in [4] where comparative graphs of H_k and σ_B versus ε for iron, copper, aluminum, and lead at 300° K were presented.

The function $\sigma_B(\dot{\varepsilon}, T)$ for mild steel was studied in [7]. The temperature range was $195° K \leq T \leq 713° K$ and deformation rate range was $0 \leq \dot{\varepsilon} \leq 10^5 sec^{-1}$. Results of the experiments which were performed with bars are presented in Figs. 1-4 for various temperatures and coordinates ($\log \dot{\varepsilon}$, $\log \sigma$) where σ is the yield stress. In Figs. 1-4 the value of $\dot{\varepsilon}$ is in sec^{-1} and σ is in kg/mm^2.

There is little data in the literature on the changes in parameters of the stressed state of metals in the deformation rate range above $10^5 sec^{-1}$. To estimate the intensity of the shear strain in this case it is possible to utilize available metal viscosity values. As was shown in [8], $\sigma_g \approx \rho \nu \dot{\varepsilon}$, where ν is the kinematic coefficient of viscosity of the metal.

Table 4 shows values of ν and $\dot{\varepsilon}$ for aluminum and steel, obtained by processing of experimental data on registration of planar cumulative flows formed by oblique impact of plates driven by a detonation wave.*

*V. I. Mali, A Study of Metal Flow under Explosive Loading, Candidates' Dissertation, Novosibirsk (1973).

TABLE 4

Steel 3		Al	
$\dot{\varepsilon}\cdot10^5$, sec^{-1}	$\nu\cdot10^{-4}$, cm^2/sec	$\dot{\varepsilon}\cdot10^{-5}$, sec^{-1}	$\nu\cdot10^{-4}$, cm^2/sec
3.3	4	7	5
7.2	3	6.9	4.6
7.5	2.5	12	3.7
15.2	1.8	21.2	3.1
27.5	1.3	39	2.9
58.5	1.3	97	1.9
137	1.2	—	—

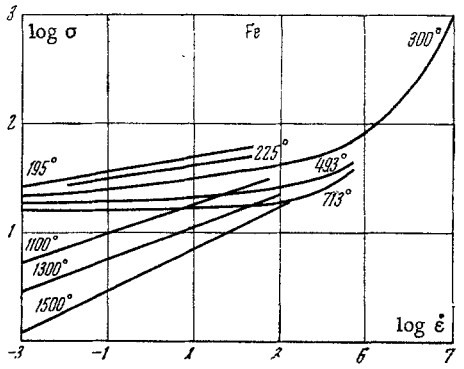

Fig. 1

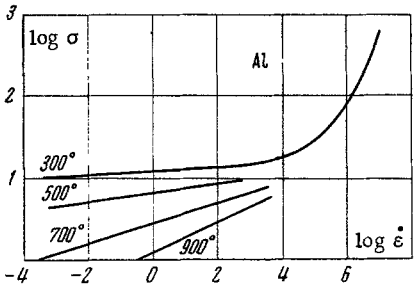

Fig. 2

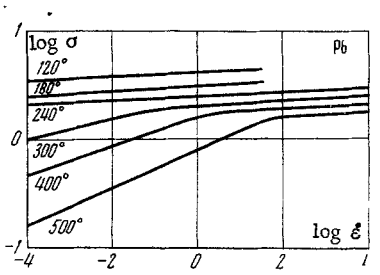

Fig. 3

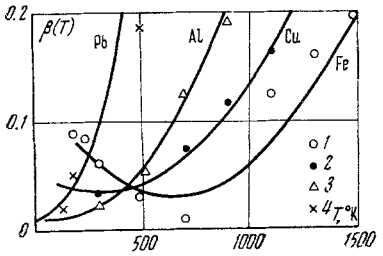

Fig. 4

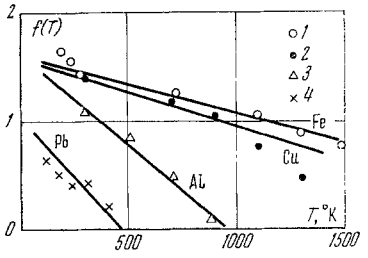

Fig. 5

Determination of viscosity coefficients is based on the assumption that decrease in flow rate in comparison to the rate predicted by accumulation theory for small values of collision angle and contact point velocity occurs because of the action of viscous forces.

Experimental results taken from [4, 6, 7, 9] are presented in the form of graphs in Figs. 1-3 for iron, aluminum, and lead.

3. The experimental results presented in Figs. 1-3 indicate that the curve $\log \sigma = f(\log \dot{\varepsilon})$ at constant temperature may be divided into two parts, each of which may be regarded as a straight line. The first portion of the deformation rate for iron ($\dot{\varepsilon} \leq 10^4$ sec^{-1}), aluminum ($\dot{\varepsilon} \leq 10^3$ sec^{-1}), copper ($\dot{\varepsilon} \leq 10^5$ sec^{-1}), lead ($\dot{\varepsilon} \leq 10$ sec^{-1}) is described satisfactorily by the formula

$$\dot{\varepsilon} = \left(\frac{\sigma}{\rho c_0^2 q}\right)^{1/\beta(T)} \exp\left(-f(T)/\beta(T)\right)$$

$$\beta(T) = n_0\left(\frac{T}{\theta_0} - n_1\right)^2 + n_2 = n^{-1}(T), \quad f(T) = F_0 - F_1\frac{T}{\theta_0}$$

Figures 4, 5 show the functions $\beta(T)$ and $f(T)$. Points on the curve correspond to experimental values for: 1) iron; 2) copper; 3) aluminum; 4) lead.

The high scattering of the experimental data of Fig. 5 for iron is due to variations in the experimental methods used and differences in the material employed.

To describe $\dot{\varepsilon}$ as a function of σ in the second interval (for iron $\dot{\varepsilon} > 10^4$ sec^{-1}, for aluminum $\dot{\varepsilon} > 10^3$ sec^{-1}, for lead $\dot{\varepsilon} > 10$ sec^{-1}) in Eq. (1.1) we introduce a correction factor of the form $\exp(-\Phi(\sigma)/RT/\mu)$, where $\Phi(\sigma)$ is determined from Eq. (1.1).

The form of the correction factor is determined by considerations of the general form of the function $\dot{\varepsilon} = \dot{\varepsilon}(\sigma, T)$, mentioned in section 1. In the absence of data on the temperature dependence of $\dot{\varepsilon}(\sigma)$ in the second interval for the function $\Phi(\sigma)$, it was impossible to consider temperature dependence.

No data for the second section was found for copper in the literature and thus, in Eq. (1.1) for U in the case of copper no correction factor was calculated. In this second interval the form of the function for lead differs from that for iron and aluminum, the slope of $\log \varepsilon = A(\log \sigma)$ in the second interval is steeper than in the first. This is accounted for in Eq. (1.1) for U by taking the sign minus instead of plus.

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